MANONMANIAM SUNDARANAR UNIVERSITY

Syllabus for Ph.D. Course Works in Mathematics

(For Affiliated Colleges and University Department)

PH.D COURSE WORKS (2016-17) ONWARDS

THE FOLLOWING ARE THE LIST OF 14 COURSE WORKS AVAILABLE FOR SELECTION ACCORDING TO THEIR REQUIREMENTS FOR PH.D. CANDIDATES IN MATHEMATICS AND **EACH OF THEM CARRIES 4 CREDITS.**

Course 1. **Commutative algebra**

Course 2. **Advanced analysis**

Course 3. **Banach algebra and spectral theory**

Course 4. **Advanced graph theory**

Course 5. **Harmonic analysis**

Course 6. **Stochastic modeling**

Course7. **Wavelets**

Course8. **Theory of near-rings**

Course9. **Advanced calculus**

Course10. **Algebraic graph theory**

Course11. **Combinatorial theory**

Course12. **Advanced domination theory in graphs**

Course13. **Graph reconstruction theory**

Course14. **Algebraic topology**

The equivalence of these course works with courses in M.Phil / M.Sc are given in**Annexure I.**

Detailed Syllabus

Course 1. COMMUTATIVE ALGEBRA (60 hours)

Preamble: It is the study of commutative rings. The objective of the paper is to introduce algebraic structure through the modules and different types of modules and it's algebraic application. A pass in PG level algebra course is the prerequisite for this paper. Out come of this paper is to motivate students to do research in diverse fields such as homological algebra, algebraic number theory, algebraic geometry, finite fields and computational algebra.

Unit I: Rings and Ideals – Modules **(12 hours)**

Unit II: Rings and Modules fractions – Primary Decomposition **(12 hours)**

Unit III: Integral Dependence and valuations – Chain conditions **(12 hours)**

Unit IV: Noetherian Rings – Artin Rings **(12 hours)**

Unit V: Discrete valuation rings and Dedekind domains **(12 hours)**

Text Book: Content and Treatment as in Atiyah and Macdonald, Introduction to Commutative Algebra, Chapters 1 to 9.

Course 2. ADVANCED ANALYSIS (60 hours)

Preamble : The objective of this course is to understand borel measure in real and complex field. Prerequisite of this course is good knowledge of calculus, real and complex analysis, topology and measure theory.Motivation is to prepare scholars with an excellence in Lp spaces for the study of analysis. The out come of this course is to help the students to undertake further research in Fourier analysis, Harmonic analysis and Functional analysis.

Unit I : Abstract Integration : The concept of measurability – Simple functions – Elementary properties of measures – Arithmetic in $[0, \infty]$ - Integration of positive functions – Integration of complex functions – The role played by sets of measure zero**(12 hours)**.

Unit II :Positive Borel Measures : Topological preliminaries – The Riesz representation theorem – Regularity properties of Borel measures – Lebesgue measure – Continuity properties of measurable functions**(12 hours)**.

Unit III :Complex Measures : Total variation – Absolute continuity – Consequences of the Radon-Nikodym theorem – Bounded linear functions on L^p - The Riesz representation theorem**(12 hours)**.

Unit IV :H^p- Spaces : Sub-harmonic functions – The spaces H^p and N - The theorem of F. and M. Reisz – Factorization theorems – The shift operator – Conjugate functions**(12 hours)**.

Unit V : Fourier Transforms : Formal properties – The inversion theorem – The Plancherel theorem – The Banach algebra L^1.

Holomorphic Fourier Transforms : Two theorems of Paley and Wiener – Quasi-analytic classes – The Denjoy- Careman theorem**(12 hours)**.

Text Book : Content and Treatment as in Walter Rudin, Real and Complex Analysis, Third Edition, Chapters 1, 2, 6, 9, 17 and 19.

Course3. BANACH ALGEBRA AND SPECTRAL THEORY (60 hours)

Preamble: This syllabus is designed to introduce the students to the topics of Banach algebra and Hilbert spaces. Knowledge expected is to be aware of the background concepts in algebra. The students are expected to know about functionals. This will motivate the students to learn about various operators and their characteristics.

Unit I: Banach algebras – Complex Homomorphisms – Basic properties of Spectra – Symbolic Calculus**(12 hours)**.

Unit II: Differentiation - Group of invertible elements – Commutative Banach algebra – Ideals and Homomorphisms – Gelfand transforms**(12 hours)**.

Unit III: Involutions – Applications to non commutative algebra – Positive Linear Functional **(12 hours)**.

Unit IV: Bounded Operators on Hilbert spaces – Bounded Operators – A commutativity theorem – Resolution of the Identity – Spectral theorem**(12 hours)**.

Unit V: Eigen values of normal operators – Positive operators and square roots – Group of invertible operators – Characterization of V* algebra**(12 hours)**.

Text Book: Content and Treatment as in Rudin, Functional Analysis, Tata McGraw Hill, Chapters 10,11 & 12.

Course 4. ADVANCED GRAPH THEORY (60 hours)

Preamble: This course aims to introduce the learner some topics for his research in graph theory. It provides several conjectures and open problems to widen the scope of research. The pre-requisite for the course is a sound knowledge in graph theory at the post- graduate level. The outcome of the course is identification area and problems for research in graph theory.

Unit I: Dominating sets in graphs - Bounds on the domination number: in terms of order, degree, size, degree, diameter and girth**(12 hours)**.
Unit II: Product graphs and Vizing's conjecture – Domatic number - Nordhaus-Gaddum type theorems - dominating functions**(12 hours)**.
Unit III: Decompositions and colorings of a graph – Generalizations of graph decompositions**(12 hours)**.
Unit IV: Necessary conditions for the existence of a G-decomposition of a graph- **c**ycle decompositions, Vertex labelings and graceful graphs**(12 hours)**.
Unit V: Perfect graphs: The perfect graph theorem – p-critical and partitionable graphs – A polyhedral characterization of perfect graphs and p-critical graphs – The strong perfect graph conjecture (and recent theorem)**(12 hours)**.

Text Books: Content and Treatment as in

1) Teresa W. Haynes, Stephen T. Hedetniemi and Peter J. Slater, Fundamentals of Domination in graphs, Marcel Decker (1998), Section 1.2, 2.1to2.4 (For Unit I) Sections 2.6, 8.3, 9.1 and 10.1 to10.3 (for Unit II).
2) Juraj Bosak, Decompositions of graphs , Kluwar Academic Publishers, Chapters 2, 3 4, 6 and 7. (for Units III and IV).

3) Martin Charles Golumbic, Algorithmic graph theory, Academic Press, Chapter 3 (for Unit V).

Course 5. HARMONIC ANALYSIS(60 hours)

Preamble: Periodic functions play a vital role in solving many problems in Mathematics and Physics. Fourier analysis is the study of various aspects of periodicity of functions. Harmonic Analysis is a natural generalization of Fourier analysis and is significant for its mathematical aspect. The pre requisite for this course is a basic knowledge of Real and Complex analysis covered in a post graduate programme in Mathematics. The outcome of the course is to help researchers in both pure and applied mathematical fields.

Unit I: Fourier series and integrals – Definitions and easy results – The Fourier transform – Convolution – Approximate identities – Fejer's theorem – Unicity theorem – Parselval relation – Fourier Stieltjes Coefficients – The classical kernels**(12 hours)**.

Unit II: Summability – Metric theorems – Pointwise summability – Positive definite sequences – Herglotz;s theorem – The inequality of Hausdorff and Young**(12 hours)**.

Unit III: The Fourier integral – Kernels on R. The Plancherel theorem – Another convergence theorem – Poisson summation formula – Bachner's theorem – Continuity theorem**(12 hours)**.

Unit IV: Characters of discrete groups and compact groups – Bochners' theorem– Minkowski's theorem**(12 hours)**.

Unit V: Hardy spaces- Invariant subspaces – Factoring F and M. Rieza theorem – Theorems of Szego and Beuoling**(12 hours)**.

Text Book: Content and Treatment as in Henry Helson, Harmonic Analysis, Hindustan Book Agency, Chapters 1.1 to 1.9, 2.1 to 3.5 and 4.1 to 4.3.

Course 6. STOCHASTIC MODELING(60 hours)

Recap : Basics of Probability space random variable – Discrete distributions and Continuous distributions – Expectation – Conditional Expectation – Moment Generating Function – Probability Generating Function – Laplace Transform – Joint Distributions – Functions of random variables and random vectors.

Unit I : Markov chains : Transition probability matrix of a Markov chain – First step Analysis – Functional of Random walks and successive runs – classification of states – Basic Limit Theorem of Markov Chain**(12 hours)**.

Unit II : Continuous time Markov Chains : Poisson distribution and Poisson process – Distributions associated with Poisson process – Pure Birth Process – Pure Death process – Birth and Death Process – Limiting behavior of Birth and Death Process – Birth and Death Process with absorbing states**(12 hours)**.

Unit III : Renewal Phenomena : Renewal process and Related concepts – Poisson process viewed a Renewal Process – Asymptotic behavior of Renewal process**(12 hours)**.

Unit IV : Branching Process and Population Growth : Branching process – branching process and generating functions – Geometrically distributed offspring – variation on Branching process – Stochastic models of Plasmid Reproduction and Plasmid copy Number partition**(12 hours)**.

Unit V : Queueing Systems : Queueing Processes – Poisson Arrival and exponentially distributed service times – The M/G/I and M/G/8 systems – variations and extensions**(12 hours)**.

Text Book : Content and Treatment as in Howard M. Taylor and Samuel Karlin, An Introduction to Stochastic Modeling (Revised Version), Academic Press, New York, 1984.

Course7. WAVELETS (60 hours)

Preamble: Wavelet analysis has drawn much attentionfrom both mathematicians and engineers alike. The emphasis of the course is on spline wavelets and time-frequency analysis. The only pre-requisite is a basic knowledge of function theory and real analysis. The outcome of the course is to enable the learner to apply the pure mathematics in signal processing and image analysis.

Unit I :An Overview : Fourier to Wavelets – Integral Wavelets Transform and Time frequency analysis – Inversion formulas and duals – Classification of Wavelets – Multi-resolution analysis – Spines and Wavelets.
Fourier Analysis : Fourier and Inverse Fourier Transformation – Continuous Time Convolution – The delta function – Fourier Transformation of square integrable functions**(12 hours)**.
Unit II : Fourier Analysis (contd): Fourier Series – Basic Convergence Theory – Poisson Summation Formula.
Wavelet Transforms and Time Frequency Analysis : The Gabor Transforms – Short time Fourier Transforms and the uncertainty principle – The integral Wavelet Transform – Dyadic Wavelets – Inversion – Frames – Wavelet Series**(12 hours)**.
Unit III :Cardinal Spline Analysis : Cardinal Spline spaces – B-splines and their basic properties – The time scale relation and an interpolating graphical display algorithm – B-Net representations and computation of cardinal splines – Constructions of cardinal splines – constructions of spline application formulas – Construction of Spline interpolation formulas**(12 hours)**.
Unit IV :Scaling functions and Wavelets : Multi-resolution analysis – Scaling functions with finite two scale relation – Direction sum Decompositions of $L^2(R)$ - Wavelets and their duals**(12 hours)**.
Unit V :Cardinal Splines Wavelets : Interpolating splines wavelets – Compactly supported spline – Wavelets – Computation of Cardinal spline Wavelets – Euler – Frebenious Polynomials**(12 hours)**.
Orthogonal Wavelets : Examples of orthogonal Wavelets – Identification of orthogonal two scale symbols – Construction of compactly supported orthogonal wavelets**(12 hours)**.
Text Book : Content and Treatment as in Charles K. Chui, An introduction to Wavelets, Academic Press, New York, 1992.
Reference Books :

1. Chui C. K. (ed) Approximation theory and Fourier Analysis, Academic Press Boston, 1991.
2. Daribechies. I. Wavelets, CBMS-NSF Series in Appl, SIAM Philadelphia, 1992.
3. Schurnaker, L. L. Spline Functions : Basic Theory, Wiley, New York, 1981.
4. Nurnberger, G. Applications to Spline Functions, Springer Verlag, New York, 1989.

Course8. THEORY OF NEAR-RINGS(60 hours)

Preamble: The main objective of this course is to provide the knowledge about the generalized ring structures. In fact, near-ring is a natural generalization of rings in the sense that the set of all endomorphisms of a group form a ring, where the set of all mappings of a group form a near-ring. The structure of near-rings is useful in project geometry to deal about generalized field conditions.

Unit I: The elements of theory of near-rings**(12 hours)**.

Unit II: Ideal theory**(12 hours)**

Unit III: Elements of structure theory**(12 hours)**

Unit IV: Near-fields**(12 hours)**

Unit V: More classes of near-rings**(12 hours)**.

Text Book: Content and Treatment as in G. Pilz, Theory of Near-rings, North Holland, Chapters 1,2,3, 8(a), 9(a) and 9(b).

Course9. ADVANCED CALCULUS(60 hours)

Preamble: The Calculus of several variables involves many branches of Mathematics such as Partial Differential Equations, Optimization, Statistics etc. The main objective of this course is to give a thorough understanding of differentiation and integration of functions of several variables. The prerequisite is a precise knowledge of Calculus of single variable. The outcome of the course is the ability to solve problems involving several variables.

Unit I : Differentiation – Basic theorems – Partial derivatives – Derivatives – Inverse Functions**(12 hours)**.
Unit II : Implicit functions – Integration – Measure zero and Content zero – Integrable Functions**(12 hours)**.
Unit III : Fubini's theorem – Partitions of Unity – Change of Variables**(12 hours)**.
Unit IV : Integration on chains – Algebraic preliminaries – Fields and Forms –Geometric preliminaries – The fundamental theorem of Calculus**(12 hours)**.
Unit V : Manifolds – Fields and Forms on Manifolds – Stokes' theorem on Manifolds - The Volume element – The Classical theorems**(12 hours)**.

Text book :
Calculus on Manifolds by Michael Spivak, The Benjamin / CummingsPublishing Company
References :
(1) Mathematical Analysis by Tom M. Apostol, Narosa Publishing Company.
(2) Advanced Calculus by Gerald B.Folland, Pearson Publishing Company.

Course10. ALGEBRAIC GRAPH THEORY (60 hours)

Preamble: This course aims to improve the knowledge of the learner to apply algebra in graph theory. It is framed to give adequate exposure about algebraic approach to graph theory. The beginner of this course is expected to have sound understanding of graph theory and algebra at PG level. The outcome of the course is to enable the student to do qualitative research in algebraic graph theory.

Unit 1: Linear Algebra in graph theory: The spectrum of a graph – Regular graphs and line graphs - The homology of graphs**(12 hours)**.
Unit 2: Spanning trees and associated structures – Complexity – Determinant expansions**(12 hours)**.
Unit 3: Symmetry and regularity of graphs: General properties of graph automorphisms – Vertex-transitive graphs – Symmetric graphs – Trivalent symmetric graphs**(12 hours)**.
Unit 4: The Covering - graph construction – Distance-transitive graphs - The feasibility of intersection arrays**(12 hours)**.
Unit 5: The Laplacian of a graph: The Laplacian matrix – trees – representations – energy and eigenvalues – connectivity – the generalized Laplacian – Multiplicities – embedding**(12 hours)**.

Text Books:

1) **Norman Biggs,** Algebraic Graph Theory, Cambridge University Press, London, 1974. Chapters 2, 3 and 4 for Unit I, 5, 6 and 7 for Unit II,C 15, 16, 17 and 18 for Unit III, 19, 20 and 21 for Unit IV.
2) **Chris Godsil, Gordon Royle,** Algebraic Graph Theory, Springer-Verlag, New York, 2006. Chapter 13 (Sections 13.1 to 13.6, 13.9 to 13.11) for Unit V.

Course11. COMBINATORIAL THEORY(60 hours)

Preamble: This objective of this course is to develop skillsto apply the techniques of combinations and permutations for counting the number of certain configurations. The prerequisite are the basic ideas on classical algebra and trigonometry. After completing this course, the student will be able to solve problems involving the distributions of objects into cells, partitions of integers, generating functions, permutations with restrictions on relative positions, rook polynomials and Polya's theory.

Unit I: Permutations and Combinations - rule of sum and product – distributions of distinct objects – Distributions of non-distinct objects**(12 hours)**.

Unit II - Generating functions for combinations – Enumerators for permutations – Distributions of distinct objects into non-distinct cells – partitions of integers – Ferrers graph – elementary relations**(12 hours)**.

Unit III: Recurrence relations – Linear recurrence relations with constant co-efficients – solution by the technique of generating functions – a special class of non-linear difference equation - recurrence relations with two indices**(12 hours)**.

Unit IV: The principle of inclusion and exclusion – general formula – derangements – rook polynomials – permutations with forbidden positions**(12 hours)**.

Unit V: Polya's theory of counting Equivalence classes under a permutation groups – Equivalence classes of functions – Weights and inventories of functions – Polya's fundamental theorem – Generalization of Polya's theorem**(12 hours)**.

Text Book: Introduction to Combinatorial Mathematics by C.L. Liu, Chapters 1 to 5.

Course12. ADVANCED DOMINATION THEORY IN GRAPHS(60 hours)

Preamble: Domination theory in graphs is a potential area of research with many open problems. The objective of this course is to introduce various branches and recent developments in domination theory. The course is designed so that each unit introduces a new aspect of domination theory. Knowledge of a post graduate course in graph theory is a prerequisite. It helps the candidate to identify an interested area with wide scope for research.

Unit I : Dominating functions in graphs: Minus domination in graphs – signed domination in graphs – real and integer domination**(12 hours)**.

Unit II :Domination parameters of a graph: Connected domination – strong and weak domination and domination balance – the least domination number – dominating strength and weakness – set and global set domination – point-set and global point-set domination – neighbourhood numbers – neighbourhood number variations – mixed domination**(12 hours)**.

Unit III :Global domination: Some early results – global interpretations of other domination invariants – applications – concerning a characterization – sub problems**(12 hours)**.

Unit IV :Distance domination in graphs : The distance domination number – the total distance domination number – independent distance domination – the distance irredundance number – relations involving distance domination parameters**(12 hours)**.

Unit V :Topics on domination in directed graphs: Definitions – motivation – kernels in digraphs – kernels and grundy functions – solutions in digraphs – domination in digraphs – applications in game theory**(12 hours)**..

Text book : Teresa W. Haynes , Stephen T. Hedetniemi and Peter J. Slater, Domination in graphs – Advanced Topics, **Chapters :** 2, 10, 11, 12 and 15.

Course13. GRAPH RECONSTRUCTION THEORY(60 hours)

Preamble: Reconstruction Conjecture is one of the foremost and famous unsolved problems in Graph Theory. It requires deep knowledge of graph theory at PG level. The learners are expected to know some interesting classes of reconstructible graphs, some reconstructible parameters of graphs, an innovative technique used in counting lemma and the current status of the Conjecture. The outcome of the course is to enable the scholars to prove more new classes of graphs and new parameters of graphs to be reconstructible.

Unit I: Reconstruction Problem : Reconstruction Conjecture – Kelly's lemma – Counting lemma**(12 hours)**

Unit II: Edge Reconstruction Problem : Edge Reconstruction Conjecture –Greenwell theorem – Maximal Planar Graphs -Edge-recognizable domination numbers**(12 hours)**.

Unit III: Diameter of Graphs : Recognizability of graphs of Diameter two –Reconstruction of edge minimal graphs of diameter two**(12 hours)**.

Unit IV: Graph Reconstruction: Graphs with at least n-1 cards isomorphic - Unicyclic graphs – Graphs of diameter two or three – Reduction usingdiameter – Vertex switching Reconstruction**(12 hours)**

Unit V: Reconstruction of Bipartite Graphs – Reconstruction of Geodesic graphs**(12 hours)**.

Text Books / Published Papers:

1. Graphs and digraphs by Mehdi Behzad, Gary Chatrand and Linda Lesniak Foster, Wadsworth International Group, 1979.
   Unit I : Chapter 10 – Sections 10.2 to 10.15

2. Topics in graph Automorphisms and Reconstruction by Josef Lauri and Raffaele Scapellato. Cambridge University Press, 2003.
   Unit II : Chapter 8 – Section 8.6 to 8.13

3. S.K. Gupta , Pankaj Mangal , Vineet paliwal , Some work towards the proof of reconstruction conjecture, Discrete Mathematics 272 (2003) 291-296.

4. R. D. Duttona, R. C. Brighamb, C. Guia, Edge-recognizable domination numbers, Discrete Mathematics 272 (2003) 47 – 51.
   Unit III

5. Recent Advances in Graph Reconstruction by S. Monikandan and J. Balakumar, Lambert Academic Publishing, Germany, 2014.
   Unit IV : Chapter 1
   Unit V : Chapter 2 and 4.

Course14. ALGEBRAIC TOPOLOGY(60 hours)

Preamble: Algebraic topology is concerned with the construction of algebraic invariants associated to topological spaces which serve to distinguish between them. Most of these invariants are *homotopy* invariants. This course elaborates topological spaces and continuous maps between them. It demonstrates the power of topological methods in dealing with problems involving shape and position of objects and continuous mappings, and shows how topology can be applied to many areas, including geometry, analysis, group theory and physics. The outcome of the course is to the ability to pursue further studies in this and related areas of the candidate.

Unit I : The Fundamental Group : Homotopy of Paths- The Fundamental Group- Covering spaces – The Fundamental Group of the circle- Retraction and Fixed points**(12 hours)**.

Unit II : The Fundamental theorem of Algebra – The Borsuk – Ulam theorem- Deformation - Retracts and Homotopy Type – The Fundamental Group of S^n - Fundamental Group of some surfaces**(12 hours)**.

Unit III : Separation Theorem in the plane: The Jordan Separation Theorem – invariance of domain - The Jordan curve Theorem- imbedding Graphs in the plane**(12 hours)**.

Unit IV: The Selfert – van Kampen Theorem: Direct sums of abelian groups- Free product of groups- Free groups – The Selfert –van kampen Theorem – The Fundamental Group of a Wedge of circles**(12 hours)**.

Unit V: Classification of surfaces: Fundamental Groups of surfaces – Homology of surfaces – Cutting and pasting – The Classification theorem – Constructing compact surfaces**(12 hours)**.

Text Book: Treatment as in : J.R.Munkres,Topology, Second Edition, New Deihi, 2006.

Unit I: Chapter 9 (Sec 5.1-5.5), Unit II:Chapter 9 (Sec 5.6-6.0), Unit III:Chapter 10 (Sec 6.1-6.4), Unit IV: Chapter 11 (Sec 6.7-7.1), Unit V: Chapter 12 (Sec 7.4-7.8)

References:

1.Dugundiji, Topology, Allyn and Bacon, Boston, 1966.

2. W.S.Massey, Algebraic Topology – An Introduction, Springer Verlag, New York, 1975

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Annexure I

EQUIVALENCE OF Ph.D. COURSE WORKS

with Courses in M.Phil / M.Sc Programmes

Sl.No	Ph.D. Course Work	Affiliated College	University Department
1.	Commutative Algebra	Research Methodology – Commutative Algebra	Research Methodology – Commutative Algebra (DTMAC1)
2.	Advanced Analysis	Same title	--------
3.	Banach Algebra and Spectral Theory	Same title	Same Title (DTMAC2)
4.	Advanced Graph Theory	Same title	Same title (DTMAE1)
5.	Harmonic Analysis	Same title	Same title (DTMAE2)
6.	Stochastic Modeling	Same title	-----
7.	Wavelets	Same title	------
8.	Theory of Near-rings	Same title	Same title (DTMAE4)
9.	Advanced Calculus	Same title	Same Title (DTMAE5)
10.	Algebraic Graph Theory	Same title	Same title (DTMAE6)
11.	Combinatorial Theory	---	M.Sc (Mathematics) IV Semester (LMAEF)

DEPARTMENT OF MATHEMATICS

MANONMANIAM SUNDARANAR UNIVERSITY, TIRUNELVELI-12

Ph.D. Course Work Papers

(Addition of two more Papers)

Paper	Name of the Course	Credit
1	Research and Teaching Methodology	4
2	Mini Project	4

Paper:I RESEARCH AND TEACHING METHODOLOGY (60 hours)

Preamble: *The objective of the course is to study basic concepts of research and teaching methodology and its implication in the area of commutative algebra. A pass in PG level algebra course is the prerequisite for this course. Out come of this course is to qualify the students to do research in diverse fields such as homological algebra, algebraic number theory, algebraic geometry, finite fields and computational algebra.*

Unit I: **Research Methodology** : An introduction - Meaning of Research Objectives of Research - Motivation in Research - Types of Research -Research Approaches - Significance of Research - Research Methods versus Methodology – Reseearch and Scientific Method - Importance of Knowing How Research is Done - Research Process - Criteria of Good Research - Problems Encountered by Researchers in India. Defining research problem - What is a Research Problem? - Selecting the Problem - Necessity of Defining the Problem Technique Involved in Defining a Problem - An Illustration. **(10 hours)**

Unit II: Commutative algebra - Modules - Rings and Modules of fractions - Primary Decomposition

(15 hours)

Unit III: Integral Dependence and valuations - Chain conditions' **(15 hours)**

Unit IV: Noetherian Rings - Artin Rings **(10 hours)**

Unit V: **Methodology of Teaching :** Teaching - Objectives of Teaching, Phases of Teaching - Teaching Method' Lecture Method, Discussion Method, Discovery Learning, Inquiry, Problem Solving Method, Project method, Seminar - Integrating ICT in Teaching: individualised Instruction, Ways for Effective presentation with Power point - Documentation - Evaluation: Formative Summative & Continuous and Comprehensive Evaluation - Later Adolescent Psychology: Meaning, Physical, Cognitive, Emotional, Social and Moral Development - Teaching Later Adolescents . **(10 hours)**

Text Book:

1. C.R Kothari, *Research Methodology - Methods and Techniques*, Second revised Edition, New Age International Publishers, 2004, Chapters 1&2 for Unit I
2. Content and Treatment as in Atiyah and Macdonald, *Introduction to Commutative Algebra,* Addision – Wesley Publishers Company (1969) Chapters 2 to 8. (For Units 2 to 4)
3) References for Unit V
   i) Sampath, K., Panneerselvam, A. & Santhanam, S. (1984). Introduction to Educational Technology (2nd revised ed.), New Delhi : Sterling Publishers.
   ii) Sharma, S.R. (2003). Effective classroom teaching modern methods, tools & techniques. Jaipur: Mangal Deep
   iii) Vedanayam, E. G. (1989) Teaching Technology for College Teachers New York: Sterling Publishers.

DEPARTMENT OF MATHEMATICS

MANONMANIAM SUNDARANAR UNIVERSITY, TIRUNELVELI-12

Ph.D. Course Work Papers

(Addition of two more Papers)

Paper	Name of the Course	Credit
1	Fixed Point Theory and Its Applications	4
2	Graphs From Algebraic Structures	4

Fixed Point Theory and Its Applications (60 hours)

Preamble: *Solving an operator equation T(x) = y is an important issue in all branches of Mathematics. In some situations, the operator equation T(x) = y is equivalently expressed as a fixed point equation f(x) = x, for some suitable mapping f so that each solution of f(x) = x contributes at least one solution to T(x) = y. Fixed point theory is a tool to solve the fixed point equation. In this course, we study necessary and sufficient conditions to be made on the operator f and its domain to ensure at least one fixed point for the mapping f, with applications of fixed point theory. Prerequisite: Topology, Functional Analysis*

Unit I : Fixed point theory of contraction and contractive type mappings - generalization and its applications **(12 hours)**

Unit II : Fixed point theory for nonexpansive mappings **(12 hours)**

Unit III: Multivalued fixed point theory and applications to Game theory **(12 hours)**

Unit V : Brouwer's and Schauder's theorem and its applications **(12 hours)**

Unit V : Development of Fixed Point Theory : Best approximation theorems and Best proximity point theory **(12 hours)**

Text Books

1. P.V.Subrahmanyam, *Elementary Fixed Point Theorems*, Springer, 2019
2. Vasile I. Istratescu, *Fixed Point Theory : An Introduction*, Reidel Publishing Company, 1981

GRAPHS FROM ALGEBRAIC STRUCTURES (60 hours)

Preamble: *Graphs from algebraic structures started by means of the construction of Cayley graphs from finite groups. Through graphs constructed from algebraic structures, the interplay between algebra and graph theory is explored well. In this course, it is aimed to introduce zero divisor graphs, total graphs and Cayley graphs from commutative rings. Also some recent developments in this research area are also exposed. On successful completion of this course, the students can pursue their research in this topic.*

Unit I: Graphs from finite groups: an overview **(12 hours)**

Unit II: The zero-divisor graph of commutative rings: A survey **(12 hours)**

Unit III: Recent results on the annihilator graph of commutative rings **(12 hours)**

Unit IV: Total graph of commutative rings and generalizations **(12 hours)**

Unit V: Domination in graphs from commutative rings **(12 hours)**

Text Books:

Yusuf F. Zakariya, *Graphs from Finite Groups: An Overview,* Proceedings of Annual National Conference, Nigeria, 2017, for Unit I
Marco Fontana, Salah-EddineKabbaj, Bruce Olberding, Irena Swanson, *Commutative Algebra: Noetherian and Non-Noetherian Perspectives,* Springer, London (2010), (Chapter 2), for Unit II
K. S. Prasad, K. B. Srinivas, P. Harikrishnan, B. Satyanarayan, *Near rings, Near fields related topics,* World Scientific(Chapter 17)**,** for Unit III
Marco Fontana, Sophie Frisch and Sarah Glaz, *Commutative Algebra: Recent Advances in Commutative Rings, Integer-Valued Polynomials and Polynomial functions,* Springer, London (2014), (Chapter 3), for Unit IV.
Syed Tariq Rizvi, Asma Ali, Vincenzo De Filippis, *Algebra and its Applications,* Springer, (2014), (Chapter 23), for Unit V.

DEPARTMENT OF MATHEMATICS

MANONMANIAM SUNDARANAR UNIVERSITY, TIRUNELVELI-12

Ph.D. Course Work Papers

(Addition of two more Papers)

Paper	Name of the Course	Credit
1	Research Topics in Graph Theory	4

RESEARCH TOPICS IN GRAPH THEORY (60 hours)

Preamble: *This course aims to introduce the learner some topics for his research in graph theory. It provides several conjectures and open problems to widen the scope of research. The pre-requisite for the course is a sound knowledge in graph theory at the post- graduate level. The outcome of the course is identification area and problems for research in graph theory.*

Unit I : Dominating sets in graphs - Bounds on the domination number- Bounds in terms of order, degree, and packing - Bounds in terms of order and size.- Grid graphs.

(12 hours)

Unit II : Dominating functions - .Y-valued parameters - Minus and signed domination - Vizing's Conjecture-Efficiency - Complementarity. **(12 hours)**

Unit III: The Reconstruction Conjectures - some basic results - maximal planar graphs **(12 hours)**

Unit IV: Factorizations and Decompositions of graphs, Labelings of graphs

(12 hours)

Unit V: Ramsey Theory- Classical Ramsey numbers - Generalized Ramsey theory. **(12 hours)**

Text Books: Content and Treatment as in

1) Teresa W. Haynes, Stephen T. Hedetniemi and Peter J. Slater, *Fundamentals of Domination in graphs*, Marcel Decker (1998),
   Unit I : Chapters 1 and 2 , Unit II : Chapter 10

2) Josef Lauri and Raffaele Scapellato, *Topics in graph Automorphisms and Reconstruction,* Cambridge University Press, Second Edition, 2016.
   Unit III : Chapter 8

3). Chartrand and L. Lesniak, *Graphs and Digraphs*, Chapman & Hall/CRC, (Third edition) 2000.
   Unit IV: Chapter 9 (Sections 9.2 and 9.3 only)
   Unit V: Chapter 12 (Sections 12.1 and 12.2 only)

DEPARTMENT OF MATHEMATICS

MANONMANIAM SUNDARANAR UNIVERSITY, TIRUNELVELI-12

Ph.D. Course Work Papers

(ADDITION OF 6 More PAPERS)

Sl. No.	Name of the course	Credit
1.	Advanced Operations Research and Fuzzy Sets	4
2.	Centrality and Convexity in Graphs	4
3.	Extensions in Pebbling in Graphs	4
4.	Labeling Techniques in Graphs	4
5.	Pebbling in Graphs	4
6.	Queueing and Inventory Models	4

ADVANCED OPERATIONS RESEARCH AND FUZZY SETS

HOURS - 4 **CREDITS - 4**

Course Outcomes:

- ❖ To study what is Simulation and Markov decision process.
- ❖ To understand the concept of Fuzzy sets and its algebra, logical aspects and relations.

Unit I

Simulation Modeling: Monte Carlo Simulation – Types of Simulation – Elements of Discrete Event Simulation – Generation of Random Numbers – Mechanics of Discrete Simulation – Methods for Gathering Statistical Observations .

(Textbook 1-Chapter:18 (except section 18.7))

Unit II

Markovian Decision Process: Scope of the Markovian Decision Problem – The Gardner Problem – Finite Stage Dynamic Programming Model – Infinite Stage Model - Linear Programming Solution –Review of Markov chains.

(Textbook 1-Chapter:19)

Unit III

The Concept of Fuzziness: Examples - Mathematical Modelling - Some Operations on Fuzzy Sets- Fuzziness as Uncertainty. **Some Algebra of Fuzzy Sets:** Boolean Algebras And Lattices – Equivalence Relations and Partitions - Composing Mappings - Isomorphisms and Homomorphisms- Alpha Cuts – Images Of Alpha Level Sets.

(Textbook 2 - Chapter 1 and 2)

Unit IV

Fuzzy Quantities: Fuzzy Quantities - Fuzzy Numbers- Fuzzy Intervals. **Logical Aspects of Fuzzy Sets:** Classical Two Valued Logic - A Three Valued Logic- Fuzzy Logic - Fuzzy And Lukasiewicz Logics - Interval Valued Fuzzy Logic - Canonical Forms - Notes Of Probabilistic Logic.

(Textbook 2 - Chapter 3 and 4)

Unit V

Fuzzy Relations: Definitions And Examples - Binary Fuzzy Relations - Operation On Fuzzy Relation - Fuzzy Partitions - Fuzzy Relations As Chu Spaces - Approximate Reasoning – Approximate Reasoning In Expert Systems - A Simple Form Of Generalized Modus Ponens - The Compositional Rule Of Inference

(Textbook 2 - Chapter 7)

Text Books:

1. Hamdy A. Taha, *Operations Research:*An Introduction, Prentice Hall Of India Pvt. Ltd., 7thEdition, 2005.
2. Hung T. Nguyen and Elbert A. Walker, *A First Course In Fuzzy Logic*, Chapman And Hall/CRC Publication, 3rdEdition, 2006.

CENTRALITY AND CONVEXITY IN GRAPHS

HOURS - 4 **CREDITS - 4**

Course Outcomes:

- To provide an indepth knowledge of central concepts in graphs
- To explore the various applications of convexity of graphs
- To motivate the students to do research in facility location problems

UNIT- I:

Distance: Eccentricity – Center – Periphery – **Detour Distance**: Detour Distance - Detour Eccentricity – Detour Center – Detour Periphery

Text Book 1: Chapter 12

UNIT - II:

Distance: The Centroid - The median – Self Median - The Path Center – The Path Centroid- Core and Pits

Text Book 2: Chapter 2

UNIT - III:

Monophonic Distance: Monophonic Distance – Detour Monophonic Number – Upper Detour Monophonic Number – Forcing Detour Monophonic Number

Text Book 3: Chapter 6

UNIT - IV:

Triangle Free Detour Distance: Vertex-to-Vertex Triangle Free Detour Distance - Vertex-to-Clique Triangle Free Detour Distance – Algorithms to find Vertex-to-Clique Triangle Free Detour Distance.

Text Book 4: Chapter 5

UNIT - V:

Triangle Free Detour Distance: Clique-to-Vertex Triangle Free Detour Distance –– Algorithms to find Clique-to-Vertex Triangle Free Detour Distance - Clique-to-Clique Triangle Free Detour Distance

Text Book 4: Chapter 5

Text Book:

1.Introduction to Graph Theory, Gary Chartrand and Ping Zhang, (Edition 2006), Tata McGraw-Hill Publishing Company Limited, New Delhi.

2. Buckley F, Harary F, Distance In Graphs, Redwood City, Addison Wesley, 1990

3. Beril Sirmacek, Graph Theory – Advanced Algorithms and Applications, InTech, 2018

4. Keerthi Asir I, Athisayanathan S, Distance In Graphs, 2018

EXTENSIONS IN PEBBLING IN GRAPHS

HOURS - 4 **CREDITS – 4**

Course Outcome:

- ❖ To know the way of determining the pebbling numbers of various types of graphs
- ❖ To study the domination cover pebbling number.

Unit I :
Pebbling number of some graphs– 2-Pebbling Property – Diameter d Graphs.

Unit II:
t-Pebbling Number of some graphs , - t-Pebbling Number of Square of Paths - t-Pebbling Number of Squares of Cycles – 2t- pebbling property.

Unit III:
Transfer Lemma - Graham's Conjecture on product of graphs – t-Pebbling Conjecture on product of graphs – Herscovici's Conjecture on product of graphs.

Unit IV:
Domination Cover Pebbling Number for Squares of Paths – Domination Cover Pebbling Number for Squares of Cycles – Domination Cover Pebbling Number for Some Unicycles – Domination Cover Pebbling Number for Cycle(Even and Odd) Lollipop

Unit V:
Covering cover Pebbling Number - Covering Cover Pebbling Number for Square of Paths – Covering Cover Pebbling Number for Square of Cycles - Covering Cover Pebbling Number for Some Cyclic and Acyclic

Text & Reference Materials:

1. F.R.K. Chung, Pebbling in hypercubes, SIAM J. Disc. Math., 2 (4) (1989), 467-472.

2. J. A. Foster and H. S. Snevily, The 2-pebbling property and a conjecture of Graham's,Graphs and Combin. 16 (2000), 231-244.

3. T. A. Clarke, R. A. Hochberg, & G. H. Hurlbert, Pebbling in diameter two graphs and products of Paths, J. Graph Theory, 25(1997), 119-128.

4. A. Lourdusamy, C. Muthulakshmi@Sasikala and T. Mathivanan, The pebbling number of the square of an odd cycle, Sciencia Acta Xaveriana Vol. 3 (2) (2012), 21-38.

5. A. Lourdusamy, and T. Mathivanan, The t-pebbling number of the Jahangir graph J3,m, Proyecciones Journal of Mathematics, Vol 34 (2), June 2015, 161-174.

LABELING TECHNIQUES IN GRAPHS

HOURS - 4 **CREDITS - 4**

Unit I:

Definitions of Graph labeling, Definitions and Examples of dif-ferent types of graph labeling: graceful labeling, harmonious labeling, magic labeling, antimagic labeling, mean labeling, prime labeling, SD-prime label-ing, binary labeling, cordial labeling, prime cordial labeling, difference cordial labeling, product cordial labeling, divisor cordial labeling, SD-prime cordial labeling.

Unit II:

Vertex equitable labeling: path, bistar, comb graph, cycle, quadrilateral snake, $K_2 + MK_1$, $K_{1,n}$ $K_{1,n+k}$ for $1 \leq K \leq 3$, ladder, $K_{1,n}$ for $N \geq 4$, any Eulerian graph with N edges, wheel, complete graph, triangular cactus. Vertex equitable labeling of subdivision and super subdivision of graphs: comb graph, bistar graph, ladder graph, arbitrary super division of any path, arbitrary super division of cycle, quadrilateral snake.

Unit III:

Vertex equitable labeling of transformed trees: transformed tree

T_p-tree, $T\Theta \overline{K}_n$, $T\hat{o} P_n$, $T\hat{o}2P_n$, $T\hat{o} C_n$, $T\bar{o} C_n$.

Unit IV:

Vertex equitable labeling of snake related graphs: quadrilateral snake, double triangular snake, double quadrilateral snake, double alternative triangular snake, double alternative quadrilateral snake.

Unit V

Vertex equitable labeling of cycle related graphs: crown, armed crown, KC_4-snakes, the graph obtained by duplicating an arbitrary vertex of a cycle, the graph obtained by duplicating an arbitrary edge of a cycle. Vertex equitable labeling of identification of graphs.

Text Materials and References:

1. J. Gallian, A dynamic survey of graph labeling, Electronic J. Combin., 19, #DS6(2016).
2. P. Jeyanthi and A. Maheswari, Some Results on Vertex Equitable Labeling, Open Journal of Discrete Mathematics, 2(2) (2012), 51-57.
3. P.Jeyanthi and A.Maheswari, Vertex Equitable Labeling of Transformed Trees, Journal of Algorithms and Computation, 44(1) (2013), 9-20.
4. P. Jeyanthi and A. Maheswari, Vertex equitable labeling of cyclic snakes and bistar graphs, J. Sci. Res., 6(1) (2014), 79-85.
5. P. Jeyanthi, A. Maheswari and M. Vijayalaksmi, Vertex Equitable Labeling of Cycle and Star Related Graphs, Journal of Scientific Research, 7(3) (2015), 33-42.
6. P. Jeyanthi, A. Maheswari and M. Vijayalaksmi, Vertex Equitable Labeling of Double Alternate Snake Graphs, Journal of Algorithms and Computation, 45(2015), 27-34.
7. P. Jeyanthi and A. Maheswari, Vertex equitable labeling of cycle and path related graphs, Util. Math., 98, (2015), 215-226.
8. P. Jeyanthi, A. Maheswari and M. Vijayalakshmi, Vertex equitable labeling of double alternate snake graphs, J. Algorithms Comput., 46 (2015) 27-34.
9. A. Lourdusamy and M. Seenivasan, Vertex equitable labeling of graphs, Journal of Discrete Mathematical Sciences and Cryptography, 11(6) (2008), 727-735.

PEBBLING IN GRAPHS

HOURS - 4 **CREDITS - 4**

Unit I :

Graph Pebbling – Distribution – Solvability – Unsolvability - Pebbling On Some
Standard Graphs As Complete Graphs, Path, Cycle, etc. 2-Pebbling Property.

Unit II:

t-Pebbling On Some Standard Graphs As Complete Graphs, Path, Cycle,.etc.- 2t- Pebbling Property-Lemke Graphs- Demonic Graphs - Pebbling On C5*C5.

Unit III:

Transfer Lemma – Grahams Conjecture On Product Of Graphs G*H – Grahams Conjecture On Product Of Cycles – Grahams Conjecture On G*H (H Satisfies the 2- Pebbling Property).

Unit IV:

Lourdusamy's Conjecture On Product Of Graphs G*H – Loudusamy's Conjecture On Product Of Cycles – Lourdusamy's Conjecture On G*H (H Satisfies the 2-Pebbling Property).

Unit V:

Herscovici's Conjecture On Product Of Graphs G*H - Herscovici's Conjecture On Product of throne graph and complete graph– Optimal Pebbling On Graphs.

Text & Reference Materials:

1. F.R.K. Chung, *Pebbling in hypercubes*, SIAM J. Disc. Math., 2 (4) (1989), 467-472.
2. J. A. Foster and H. S. Snevily, The 2-pebbling property and a conjecture of Graham's,Graphs and Combin. 16 (2000), 231-244.
3. D.S. Herscovici and A.W. Higgins, The pebbling number of *C*5 * *C*5, *Discrete Math.,* 187(1998), 123-135.
4. A. Lourdusamy and S. Somasundaram, The t-pebbling number of graphs, South East Asian Bulletin of Mathematics, 30 (2006), 907-914.
5. D. Herscovici, Graham's pebbling conjecture on products of cycles, J. Graph Theory 42 (2003), 141-154.
6. S. Wang, Pebbling and Graham's conjecture, Disc. Math., 226(3) (2001), 6 431-438.
7. A. Lourdusamy, t-pebbling the product of graphs, Acta Ciencia Indica, XXXII (M.No.1) (2006), 171-176.
8. A. Lourdusamy, S.S. Jeyaseelan and A.P. Tharani, *t*-pebbling the product of fan graphs and the product of wheel graphs, *International Mathematical Forum,* 32 (2009), 1573 - 1585.
9. Dong-Lin Hao, Ze-Tu Gao, Jian-Hua Yin, Herscovici's Conjecture on the Product of the Thorn Graphs of the Complete Graphs, J. Oper. Res. Soc. China (2014) 2:263–269
10. Friedman, T.,Wyels, C.: Optimal pebbling of paths and cycles.Mathematics. arXiv:math.CO/0506076.

QUEUEING AND INVENTORY MODELS

HOURS - 4 **CREDITS - 4**

Objective:

- To understand the concept of Queueing theory interms of stochastic process.
- To study the behavior of Inventory Models.

Unit I

Queueing Systems: General Concepts: Introduction - Queueing Processes – Notation – Transient and Steady State Behavior – Limitations of the Steady State Distribution - Some General Relationships in Queueing Theory – Poisson Arrival Process and Its Characteristics.

(Textbook 1-Chapter: 2)

Unit II

Birth and Death Queueing Systems: Exponential Models: Introduction – The Simple M/M/1 Queue – System with Limited Waiting Space: The M/M/1/K Model – Birth and Death Processes: Exponential Models – The M/M/∞ Model: Exponential Model with an Infinite Number of Servers – The Model M/M/c – The M/M/c System: Erlang Loss Model.

(Textbook 1 - Chapter: 3 (3.1-3.7))

Unit III

Non-Birth and Death Queueing Systems: Markovian Models: Introduction – Bulk Queues – Queueing Models with Bulk (Batch) Service – M/M(a,b)/1: Transient State Distribution – Two Server Model: M/M(a,b)/2 – The M/M(1,b)/c Model. **Network of Queues:** Network of Markovian Queues – Channels in Series or Tandem Queues – Jackson Network – Closed Markovian Network – Cyclic Queue – BCMP Networks.

(Textbook 1- Chapter: 4 and 5)

Unit IV

Inventory Theory: Components of Inventory Models – Deterministic Continuous Review Models – A Deterministic Periodic Review Model.

(Textbook 2 - Chapter: 19 (19.2-19.4))

Unit V

A Stochastic Continuous Review Model – A Stochastic Single Period Model for Perishable Products – Stochastic Periodic Review Models.

(Textbook 2 - Chapter: 19 (19.5-19.7))

Text Book:

1) Medhi J, *Stochastic Models in Queueing Theory*, Academic Press, Second Edition, 2003.

2) Frederick S. Hillier, Gerald J. Lieberman., *Introduction to Operations Research*, McGraw-Hill Higher Education, Seventh Edition, 2001.